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Addendum: The SNO Solar Neutrino Data, Neutrinoless Double Beta-Decay and Neutrino Mass Spectrum

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Abstract

We update our earlier study in [1], which was inspired by the 2002 SNO data, on the implications of the results of the solar neutrino experiments for the predictions of the effective Majorana mass in neutrinoless double beta-decay, $|{<}m{>}|$. We obtain predictions for $|{<}m{>}|$ using the values of the neutrino oscillation parameters, obtained in the analyzes of the presently available solar neutrino data, including the just published data from the salt phase of the SNO experiment, the atmospheric neutrino and CHOOZ data and the first data from the KamLAND experiment. The main conclusion reached in [1] of the existence of significant lower bounds on $|{<}m{>}|$ in the cases of neutrino mass spectrum of inverted hierarchical (IH) and quasi-degenerate (QD) type is strongly reinforced by fact that combined solar neutrino data i) exclude the possibility of $\cos 2\theta_{\odot} = 0$ at more than 5 s.d., ii) determine as a best fit value $\cos 2\theta_{\odot} = 0.40$, and ii) imply at 95% C.L. that $\cos 2\theta_{\odot} \gtrsim 0.22$, $\theta_{\odot}$ being the solar neutrino mixing angle. For the IH and QD spectra we get using, e.g., the 90% C.L. allowed ranges of values of the oscillation parameters, $|{<}m{>}| \gtrsim 0.010$ eV and $|{<}m{>}| \gtrsim 0.043$ eV, respectively. We also comment on the possibility to get information on the neutrino mass spectrum and on the CP-violation in the lepton sector due to Majorana CP-violating phases.

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1 Introduction

In the present article we investigate the implications of the recently published data from the salt phase of measurements of the SNO solar neutrino experiment [2] for the predictions of the effective Majorana mass in neutrinoless double beta decay, $|<m>|$ (see, e.g., [3, 4, 5]). We update our earlier similar analysis in [1], which was inspired by the 2002 SNO data [6]. As is well known, the neutrinoless double beta ($(\beta\beta)_{0\nu}-$) decay, $(A,Z) \rightarrow (A,Z+2) + e^- + e^-$, is allowed if neutrino mixing, involving the electron neutrino ν_e, is present in the weak charged lepton current and the neutrinos with definite mass are Majorana particles (see, e.g., [3]). Strong evidences for neutrino mixing, i.e., for oscillations of solar electron neutrinos ν_e driven by nonzero neutrino masses and neutrino mixing [7], have been obtained in the solar neutrino experiments [8, 9]: the pioneering Davis et al. (Homestake) experiment [10, 11] and in Kamiokande, SAGE, GALLEX/GNO and Super-Kamiokande. These evidences have been spectacularly reinforced during the last two years by the data from the SNO solar neutrino and KamLAND reactor antineutrino experiments. Under the assumption of CPT-invariance, the observed disappearance of reactor $\bar{\nu}_e$ in the KamLAND experiment [12] confirmed the interpretation of the solar neutrino data in terms of $\nu_e \rightarrow \nu_{\mu,\tau}$ oscillations, induced by nonzero neutrino masses and nontrivial neutrino mixing. The KamLAND results practically established the large mixing angle (LMA) MSW solution as unique solution of the solar neutrino problem.

The combined 2-neutrino oscillation analysis of the solar neutrino and KamLAND data identified two distinct solution sub-regions within the LMA solution region - LMA-I,II (see, e.g., [13, 14]). The best fit values of the two-neutrino oscillation parameters - the solar neutrino mixing angle $\theta_\odot$ and the mass squared difference $\Delta m^2_\odot$, in the two sub-regions - LMA-I and LMA-II, read (see, e.g., [13]): ${\Delta m^2_\odot}^{I} = 7.3 \times 10^{-5}$ eV^2, ${\Delta m^2_\odot}^{II} = 1.5 \times 10^{-4}$ eV^2, and $\tan^2\theta^I_\odot = \tan^2\theta^{II}_\odot = 0.46$. The LMA-I solution was preferred statistically by the data. At 90% C.L. it was found in, e.g., [13]:

$$\Delta m^2_\odot \cong (5.6 - 17) \times 10^{-5} \text{ eV}^2, \quad \tan^2\theta_\odot \cong (0.32 - 0.72) \; . \tag{1}$$

Very recently the SNO collaboration published data from the salt phase of the experiment [2]. For the ratio of the CC and NC event rates, in particular, the collaboration finds: $R_{CC/NC} = 0.306 \pm 0.026 \pm 0.024$. Adding the statistical and the systematic errors in quadratures one gets at 3 s.d.: $R_{CC/NC} \leq 0.41$. As was shown in [15], an upper limit of $R_{CC/NC} < 0.5$ implies a significant upper limit on $\Delta m^2_\odot$ smaller than 2×10^{-4} eV^2. In the case of interest, $R_{CC/NC} \leq 0.41$, one finds using the results from [15]: $\Delta m^2_\odot \lesssim 1.5 \times 10^{-4}$ eV^2. Thus, the new SNO data on $R_{CC/NC}$ implies stringent constraints on the LMA-II solution. A combined statistical analysis of the data from the solar neutrino and KamLAND experiments, including the latest SNO results, shows [16] that the LMA-II solution is allowed only at 99.13% C.L. Furthermore, the data have substantially reduced the maximal allowed value of $\sin^2\theta_\odot$, excluding the possibility of maximal mixing [2] at 5.4 s.d. This has very important implications for the predictions of $|<m>|$ [1, 18, 19].

A 3-ν oscillation analyzes of all available data, including the latest SNO results, were performed in [16, 20] [3]. It was found, in particular, in [16] that at 90% C.L.,

$$0.23 \lesssim \sin^2\theta_\odot \lesssim 0.38 \quad \text{for} \quad \sin^2\theta = 0.0, \tag{2}$$

$$0.25 \lesssim \sin^2\theta_\odot \lesssim 0.36 \quad \text{for} \quad \sin^2\theta = 0.04, \tag{3}$$

[2]The future data from SNO on the day-night effect and the spectrum of e^- from the CC reaction, and the future high statistics data from KamLAND, in principle, can resolve completely the LMA-I - LMA-II solution ambiguity and can constrain further the solar neutrino mixing angle (see [15, 17, 16] and the references quoted therein).

[3]Combined 2-neutrino oscillation analyzes of the solar neutrino and KamLAND data were completed earlier in refs. [21], and in [22].

where θ is the angle limited by the CHOOZ and Palo Verde experiments [23]. The best fit (BF) values in both cases read $\sin^2\theta_\odot = 0.30$. The allowed values of $\Delta m^2_\odot$ in the LMA-I region have not changed considerably and are given at 90% C.L. by [16]:

$$5.6 \times 10^{-5}\ \text{eV}^2 \lesssim \Delta m^2_\odot \lesssim 9.2 \times 10^{-5}\ \text{eV}^2 \quad \text{for} \quad \sin^2\theta = 0.0, \tag{4}$$
$$6.1 \times 10^{-5}\ \text{eV}^2 \lesssim \Delta m^2_\odot \lesssim 8.5 \times 10^{-5}\ \text{eV}^2 \quad \text{for} \quad \sin^2\theta = 0.04. \tag{5}$$

The $\Delta m^2_\odot$ best fit value is practically the same for the two values of $\sin^2\theta$: $\Delta m^2_\odot \cong 7.2\times10^{-5}\ \text{eV}^2$.

There are also strong evidences for oscillations of the atmospheric ν_μ ($\bar{\nu}_\mu$) from the observed Zenith angle dependence of the multi-GeV μ-like events in the SuperKamiokande experiment [24]. The combined analysis [25] of atmospheric neutrino data and the data from the K2K long base line accelerator experiment [26], shows that at 90% C.L. the neutrino mass squared difference responsible for the atmospheric neutrino oscillations lies in the interval

$$2.0 \times 10^{-3}\ \text{eV}^2 \lesssim \Delta m^2_{\rm A} \lesssim 3.2 \times 10^{-3}\ \text{eV}^2. \tag{6}$$

The $\Delta m^2_{\rm A}$ best fit value found in [25] reads: $\Delta m^2_{\rm A}|_{\rm BF} = 2.6 \times 10^{-3}\ \text{eV}$. Let us note that the preliminary results of an improved analysis of the SK atmospheric neutrino data, performed recently by the SK collaboration, gave [24]

$$1.3 \times 10^{-3}\,\text{eV}^2 \lesssim |\Delta m^2_{\rm A}| \lesssim 3.1 \times 10^{-3}\,\text{eV}^2\,, \qquad 90\%\ \text{C.L.}, \tag{7}$$

with best fit value $|\Delta m^2_{\rm A}| = 2.0 \times 10^{-3}\ \text{eV}^2$. Adding the K2K data [26], the authors [27] find the same best fit value and

$$1.55 \times 10^{-3}\,\text{eV}^2 \lesssim |\Delta m^2_{\rm A}| \lesssim 2.60 \times 10^{-3}\,\text{eV}^2\,, \qquad 90\%\ \text{C.L.} \tag{8}$$

The last parameter relevant for our further discussion is the neutrino mixing angle θ, limited by CHOOZ and Palo Verde experiments. The precise limit on θ is $\Delta m^2_{\rm A}$ – dependent (see, e.g, [28]). For the values of $\Delta m^2_{\rm A}$ found in [25] (see eq. (6)), the upper bound on $\sin^2\theta$ at 90% (99.73%) C.L. reads: $\sin^2\theta < 0.03$ (0.05). Using the latest SK preliminary result, one gets at 90% (99.73%) C.L. from a combined 3-ν oscillation analysis of the solar neutrino, CHOOZ and KamLAND data [16]:

$$\sin^2\theta < 0.047\ (0.074). \tag{9}$$

Under the assumptions of 3-neutrino mixing, for which we have compelling evidences from the experiments with solar and atmospheric neutrinos and from the KamLAND experiment, of massive Majorana neutrinos and of $(\beta\beta)_{0\nu}$-decay generated *only by the (V-A) charged current weak interaction via the exchange of the three Majorana neutrinos* ν_j, the effective Majorana mass in $(\beta\beta)_{0\nu}$-decay of interest is given by (see, e.g., [3, 5]):

$$|{<}m{>}| = \left|m_1|U_{\rm e1}|^2 + m_2|U_{\rm e2}|^2\, e^{i\alpha_{21}} + m_3|U_{\rm e3}|^2\, e^{i\alpha_{31}}\right|\,, \tag{10}$$

where U_{ej}, $j = 1,2,3$, are the elements of the first row of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix [29] U, $m_j > 0$ is the mass of the Majorana neutrino ν_j, and α_{21} and α_{31} are two Majorana CP-violating phases [30, 31]. One can express [32] the masses $m_{2,3}$ and the elements U_{ej} respectively in terms of m_1, $\Delta m^2_\odot$ $\Delta m^2_{\rm A}$, and of $\theta_\odot$, θ (see further).

In ref. [1] we have analyzed the implications of the results of the solar neutrino experiments, including the 2002 SNO data, which favored the LMA MSW solution of the solar neutrino problem with $\tan^2\theta_\odot < 1$, for the predictions of $|{<}m{>}|$. Neutrino mass spectra with normal mass hierarchy, with inverted hierarchy and of quasi-degenerate type were considered (see, e.g., [5]). From the fact that $\cos 2\theta_\odot \geq 0.26$, which followed (at 99.73% C.L.) from the analysis of the solar

neutrino data performed in [6], we found significant lower bounds on $|<m>|$ in the cases of neutrino mass spectrum of quasi-degenerate and inverted hierarchical type: $|<m>| \gtrsim 0.03$ eV and $|<m>| \gtrsim 8.5 \times 10^{-3}$ eV, respectively. We have also found that if the neutrino mass spectrum were hierarchical (with inverted hierarchy), $|<m>|$ were limited from above: $|<m>| \lesssim 8.2 \times 10^{-3}$ (8.0×10^{-2}) eV. These results led us to conclude that a measured value of $|<m>| \gtrsim 10^{-2}$ eV could provide fundamental information on the type of the neutrino mass spectrum. It was also shown that such a result could provide a significant upper limit on the mass of the lightest neutrino m_1 as well. In refs. [33, 34] the possibilities to determine the type of neutrino mass spectrum and to get information of CP-violation in the lepton sector, associated with Majorana neutrinos from a measurement of $|<m>|$ have been studied in greater detail. In these articles, in particular, the uncertainty in the determination of $|<m>|$ due to an imprecise knowledge of the relevant nuclear matrix elements and prospective experimental errors in the measured values of the neutrino oscillation parameters, entering into the expression of $|<m>|$, were taken into account. In [33] the results of analyzes of the solar and atmospheric neutrino and the first KamLAND data were used in obtaining predictions for $|<m>|$ [4].

In the present Addendum, we update the predictions for $|<m>|$ derived in ref. [1] and the conclusion reached in the indicated article by taking into account the implications of the recently announced data from the salt phase measurements of the SNO experiment. We also comment very briefly how the results obtained in [33, 34] are modified.

2 Predictions for the Effective Majorana Mass Parameter $|<m>|$

The predicted value of $|<m>|$ depends in the case of $3-\nu$ mixing on the oscillation parameters $\Delta m^2_{\rm A}$, $\theta_\odot$, $\Delta m^2_\odot$ and θ (see, e.g., [5]). Following [1], we will use the convention in which $m_1 < m_2 < m_3$. In this convention one has $\Delta m^2_{\rm A} \equiv \Delta m^2_{31}$. In what regards $\Delta m^2_\odot$, there are two possibilities: $\Delta m^2_\odot \equiv \Delta m^2_{21}$, or $\Delta m^2_\odot \equiv \Delta m^2_{32}$. The former corresponds to neutrino mass spectrum with normal hierarchy, while the latter - to neutrino mass spectrum with inverted hierarchy. Obviously, $m_3 = \sqrt{m_1^2 + \Delta m^2_{\rm A}}$. If $\Delta m^2_\odot \equiv \Delta m^2_{21}$, one has $m_2 = \sqrt{m_1^2 + \Delta m^2_\odot}$ and the following relations are valid: $|U_{\rm e1}|^2 = \cos^2\theta_\odot(1 - |U_{\rm e3}|^2)$, $|U_{\rm e2}|^2 = \sin^2\theta_\odot(1 - |U_{\rm e3}|^2)$, and $|U_{\rm e3}|^2 \equiv \sin^2\theta$. In the case of neutrino mass spectrum with inverted hierarchy, we have $m_2 = \sqrt{m_1^2 + \Delta m^2_{\rm A} - \Delta m^2_\odot}$ and $|U_{\rm e2}|^2 = \cos^2\theta_\odot(1 - |U_{\rm e1}|^2)$, $|U_{\rm e3}|^2 = \sin^2\theta_\odot(1 - |U_{\rm e1}|^2)$, $|U_{\rm e1}|^2 \equiv \sin^2\theta$. Given $\Delta m^2_\odot$, $\Delta m^2_{\rm A}$, $\theta_\odot$ and $\sin^2\theta$, the value of $|<m>|$ depends strongly on the type of the neutrino mass spectrum and on the value of the lightest neutrino mass, m_1, as well as on the values of the two Majorana CP-violation phases present in the PMNS matrix, α_{21} and α_{31} (see eq. (10)).

As in ref. [1], we consider the predictions of $|<m>|$ for the following three specific types of neutrino mass spectrum: i) normal *hierarchical* (NH), corresponding to a spectrum with normal hierarchy and $m_1 \ll m_{2,3}$, ii) inverted *hierarchical* (IH), characterized by inverted hierarchy and $m_1 \ll m_{2,3}$, and iii) quasi-degenerate spectrum (QD) which is realized if $m_1 \cong m_2 \cong m_3 \equiv m_0$ and $m^2_{1,2,3} \gg \Delta m^2_{\rm A}, \Delta m^2_\odot$. Let us note that in the case of the QD spectrum, $|<m>|$ is essentially independent of $\Delta m^2_{\rm A}$ and $\Delta m^2_\odot$, and, as long as the Majorana CP-violation phases α_{21} and α_{31} are not constrained, the two possibilities, $\Delta m^2_\odot \equiv \Delta m^2_{21}$ and $\Delta m^2_\odot \equiv \Delta m^2_{32}$, lead *effectively* to the same predictions for the allowed range of values of $|<m>|$.

In Tables 1 and 2 we give the i) maximal predicted value of $|<m>|$ in the case of NH neutrino mass spectrum, ii) the minimal and maximal values of $|<m>|$ for the IH spectrum, and iii) the

[4] A concise discussion of the relevant formalism, of the status of the searches for $(\beta\beta)_{0\nu}$-decay and of the physics potential of the $(\beta\beta)_{0\nu}$-decay experiments as well as of the predictions for $|<m>|$ before the publication of the latest SNO data, can be found in [35].

minimal value of $|{<}\,m\,{>}|$ for the QD spectrum. The indicated values of $|{<}\,m\,{>}|$ are calculated for the best fit values and for the 90% C.L. allowed ranges of $\Delta m^2_{\rm A}$ from [25] and [27], and of $\sin^2\theta_\odot$ and $\Delta m^2_\odot$ in the LMA-I solution region, obtained in the analysis in ref. [16]. In Figs. 1 and 2

$\sin^2\theta$	$\|{<}\,m\,{>}\|^{\rm NH}_{\rm max}$	$\|{<}\,m\,{>}\|^{\rm IH}_{\rm min}$	$\|{<}\,m\,{>}\|^{\rm IH}_{\rm max}$	$\|{<}\,m\,{>}\|^{\rm QD}_{\rm min}$
0.0	2.6 (2.6)	19.9 (17.3)	50.5 (44.2)	79.9
0.02	3.6 (3.5)	19.5 (17.0)	49.5 (43.3)	74.2
0.04	4.6 (4.3)	19.1 (16.6)	48.5 (42.4)	68.5

Table 1: The maximal values of $|{<}\,m\,{>}|$ (in units of 10^{-3} eV) for the NH and IH spectra, and the minimal values of $|{<}\,m\,{>}|$ (in units of 10^{-3} eV) for the IH and QD spectra, for the best fit values of the oscillation parameters and $\sin^2\theta = 0.0$, 0.02 and 0.04. The results for the NH and IH spectra are obtained for $\Delta m^2_{\rm A}\,|_{\rm BF} = 2.6\times 10^{-3}$ eV ^{2} (2.0×10^{-3} eV ^{2} $-$ values in brackets) and $m_1 = 10^{-4}$ eV, while those for the QD spectrum correspond to $m_0 = 0.2$ eV.

$\sin^2\theta$	$\|{<}\,m\,{>}\|^{\rm NH}_{\rm max}$	$\|{<}\,m\,{>}\|^{\rm IH}_{\rm min}$	$\|{<}\,m\,{>}\|^{\rm IH}_{\rm max}$	$\|{<}\,m\,{>}\|^{\rm QD}_{\rm min}$
0.0	3.7 (3.7)	10.1 (8.7)	56.3 (50.6)	47.9
0.02	4.7 (4.6)	9.9 (8.6)	55.1 (49.6)	42.8
0.04	5.5 (5.3)	11.4 (9.9)	54.0 (48.6)	45.4

Table 2: The same as in Table 1 but for the 90% C.L. allowed values of $\Delta m^2_\odot$ and $\theta_\odot$ obtained in [16], and of $\Delta m^2_{\rm A}$ given in eq. (6) (eq. (8) - results in brackets).

we show the allowed ranges of values of $|{<}\,m\,{>}|$ as a function of m_1 for the cases of spectrum with normal and inverted hierarchy. The predictions for $|{<}\,m\,{>}|$ are obtained by using the best fit (Fig. 1), and the allowed at 90% C.L. (Fig. 2), values of $\Delta m^2_\odot$, $\theta_\odot$ and $\Delta m^2_{\rm A}$ from refs. [16] and [27] and for three fixed values of $\sin^2\theta$.

Let us recall that $|{<}\,m\,{>}|^{\rm IH,QD}_{\rm min}$ are given approximately by [1] (see also [18, 19]):

$$|{<}\,m\,{>}|^{\rm IH}_{\rm min} \cong \sqrt{\Delta m^2_{\rm A}}\ \cos^2\theta\ |\cos 2\theta_\odot|, \tag{11}$$

$$|{<}\,m\,{>}|^{\rm QD}_{\rm min} \cong m_0 \left|\cos^2\theta\ \cos 2\theta_\odot - \sin^2\theta\right|. \tag{12}$$

According to [27, 16], we have at 99.73% C.L.: $\Delta m^2_{\rm A} \gtrsim 1.1\times 10^{-3}$ eV^2 and $\sin^2\theta < 0.074$. The combined analysis of the solar neutrino data, including the latest SNO results, shows [2, 16] that i) the possibility of $\cos 2\theta_\odot = 0$ is excluded at more than 5 s.d., ii) the best fit value of $\cos 2\theta_\odot$ is $\cos 2\theta_\odot = 0.40$, and iii) that at 95% C.L. one has for $\sin^2\theta = 0$ (0.04), $\cos 2\theta_\odot \gtrsim 0.22$ (0.24). These new results firmly establish the existence of significant lower bounds on $|{<}\,m\,{>}|$ in the cases of IH and QD neutrino mass spectra. That has fundamental implications for the searches for $(\beta\beta)_{0\nu}$-decay.

A comparison of the results for $|{<}\,m\,{>}|$, obtained using the best fit values of $\Delta m^2_\odot$, $\theta_\odot$ and $\Delta m^2_{\rm A}$, with those reported in refs. [1, 34] shows that the predictions for $|{<}\,m\,{>}|$ did not change considerably. This is a consequence of the fact that the best fit values of $\Delta m^2_\odot$ and $\sin^2\theta_\odot$ are not very different from the values used as input in refs. [1, 34]. With the recent preliminary result $\Delta m^2_{\rm A}\,|_{\rm BF} = 2.0\times 10^{-3}$ eV ^{2} of the improved analysis of the SK atmospheric neutrino data [24] one gets somewhat smaller values of $|{<}\,m\,{>}|$ in the case of IH spectrum. For $\sin^2\theta \sim 0$, for instance,

one finds 17×10^{-3} eV $\lesssim |<\!m\!>|^{\rm IH} \lesssim 44 \times 10^{-3}$ eV . The ranges of $|<\!m\!>|$ in the NH and QD spectra depend weakly on $\Delta m^2_{\rm A}$ in this case [5] and therefore $|<\!m\!>|^{\rm NH}_{\rm max}$ and $|<\!m\!>|^{\rm QD}_{\rm min}$ reported in Table 1 are essentially the same as the ones reported in refs. [1, 34].

According to the 3-ν combined analysis of the solar neutrino and KamLAND and CHOOZ data [16], for $\sin^2\theta = 0$ (0.04), the lower bound on $\cos 2\theta_\odot$ at 90% C.L. reads: $\cos 2\theta_\odot|_{\rm MIN} \gtrsim 0.24$ (0.28). Therefore the main conclusion that in the case of the IH and the QD spectra there exist significant lower bounds on $|<\!m\!>|$ [1], not only still holds, but is considerably strengthened, as we have already emphasized. More specifically, in the case of IH spectrum we get for $\sin^2\theta = 0$ (0.04) $|<\!m\!>|^{\rm IH}_{\rm min} = 0.010$ (0.011) eV if we use eq. (6), and $|<\!m\!>|^{\rm IH}_{\rm min} = 0.0087$ (0.0099) eV utilizing the preliminary result given in eq. (8).

In the case of QD neutrino mass spectrum, a larger lower bound on $\cos 2\theta_\odot$ implies a larger value of $|<\!m\!>|^{\rm QD}_{\rm min}$, which now reads: $|<\!m\!>|^{\rm QD}_{\rm min} \sim 0.043 - 0.048$ eV . This should be compared with the value found in [34]: $|<\!m\!>|^{\rm QD}_{\rm min} \sim 0.03$ eV .

Let us note that for the presently allowed at 90% C.L. values of $\Delta m^2_\odot$, $\theta_\odot$ and $\Delta m^2_{\rm A}$, and for $\sin^2\theta \gtrsim 0.03$, there exists a lower bound on $|<\!m\!>|$ in the case of NH spectrum, provided $m_1 << 10^{-3}$ eV: one finds $|<\!m\!>|^{\rm NH}_{\rm max} \gtrsim {\rm few} \times 10^{-4}$ eV . A complete cancellation of the different terms contributing to $|<\!m\!>|$ is allowed in this case only if

$$\sin^2\theta > \sqrt{\frac{(\Delta m^2_\odot\,)_{\rm MIN}}{(\Delta m^2_{\rm A}\,)_{\rm MAX}}}(\sin^2\theta_\odot)_{\rm MIN}. \tag{13}$$

Given the 90% C.L. allowed ranges of the oscillation parameters entering into the above inequality, the "cancellation" condition can be satisfied for $\sin^2\theta > 0.038$. Lower allowed values of $\Delta m^2_{\rm A}$ or a more stringent upper limit on $\sin^2\theta$ would further strengthen this result. It should be emphasized, however, that one can have $|<\!m\!>| \cong 0$ even for $\sin^2\theta \lesssim 0.030$ if m_1 is sufficiently large (see Figs. 1 and 2). More generally, since at present the two Majorana CP-violation phases α_{21} and α_{31} are not constrained and the existing upper bounds on m_1 are not sufficiently stringent, one can always have $|<\!m\!>| = 0$ in the case of neutrino mass spectrum with normal hierarchy [18].

The 95% C.L. allowed ranges of $\Delta m^2_\odot$ and $\theta_\odot$ differ only marginally from those derived at 90% C.L., while the upper limit on $\sin^2\theta$ changes from 0.047 to 0.053 [16]. The intervals of allowed values of $\Delta m^2_{\rm A}$ at 95% C.L., according to the older and the latest analyzes [25] and [27] do not differ considerably from the 90% C.L. ones, eqs. (6) and (8), and are given respectively by $\Delta m^2_{\rm A} = (1.8 - 3.4) \times 10^{-3}$ eV^2 and $\Delta m^2_{\rm A} = (1.4 - 2.8) \times 10^{-3}$ eV^2. These results imply that the predictions for $|<\!m\!>|$, obtained using the 95% C.L. allowed ranges of the neutrino oscillation parameters, differ insignificantly from the predictions based on the 90% C.L. ranges of the parameters, which are illustrated in Table 2 and Fig. 2.

Similar conclusion about the existence of a significant and robust lower bound on $|<\!m\!>|$, $|<\!m\!>| \gtrsim 10^{-2}$ eV, in the case of IH neutrino mass spectrum, has been reached also in [36], where a χ^2-method of analysis was employed.

3 Constraining the Type of Neutrino Mass Spectrum and/or CP-violation Associated with Majorana Neutrinos

In refs. [33, 34] the possibilities to discriminate between the different types of neutrino mass spectrum and to get information about CP-violation associated with Majorana neutrinos by means of

[5]The effective Majorana mass $|<\!m\!>|$ for the NH spectrum practically does not depend on $\Delta m^2_{\rm A}$ if $\sin^2\theta$ is sufficiently small, so that $\sqrt{\Delta m^2_\odot}\,\sin^2\theta_\odot \gg \sqrt{\Delta m^2_{\rm A}}\,\sin^2\theta$. Even for $\sin^2\theta$ close to its 90% C.L. upper limit of 0.047, the change in $|<\!m\!>|^{\rm NH}_{\rm max}$ due to the lower value of $\Delta m^2_{\rm A}\,|_{\rm BF}$ amounts at most to 6 %.

a measurement of $|{<}m{>}|$ were studied. The uncertainty in the relevant nuclear matrix elements and prospective experimental errors in the values of the oscillation parameters, in $|{<}m{>}|$, and for the case of QD spectrum - in m_1, were taken into account. Here we update the results of [33, 34] using the same notations. We denote by $(|{<}m{>}|_{\rm exp})_{\rm MIN}$ the value of $|{<}m{>}|$ obtained from a measurement of the $(\beta\beta)_{0\nu}$-decay half life-time of a given nucleus by using the largest physical nuclear matrix element, and by ζ the "theoretical uncertainty" in $|{<}m{>}|$ due to the imprecise knowledge of the nuclear matrix element. Thus, an experiment measuring a $(\beta\beta)_{0\nu}$-decay half-life time will determine a range of values of $|{<}m{>}|$ corresponding to

$$(|{<}m{>}|_{\rm exp})_{\rm MIN} \; - \Delta \leq |{<}m{>}| \; \leq \zeta((|{<}m{>}|_{\rm exp})_{\rm MIN} \; + \Delta), \tag{14}$$

where Δ denotes the experimental error in the measurement of $|{<}m{>}|$. A part of the analysis in [33, 34] was performed by using the best fit values of the solar and the atmospheric neutrino oscillation parameters and including 1 s.d. (3 s.d.) uncertainties of 5% (15%) on $\tan^2\theta_\odot$ and $\Delta m^2_\odot$, and of 10% (30%) on $\Delta m^2_{\rm A}$. We follow [33, 34] in the choice for the ranges of the indicated parameters. In Table 3 we report the predicted i) maximal value of $|{<}m{>}|$ in the case of NH spectrum, ii) the minimal and maximal values of $|{<}m{>}|$ for the IH neutrino mass spectrum, and iii) the minimal value of $|{<}m{>}|$ for the QD spectrum.

$\sin^2\theta$	$\|{<}m{>}\|^{\rm NH}_{\rm max}$	$\|{<}m{>}\|^{\rm IH}_{\rm min}$	$\|{<}m{>}\|^{\rm IH}_{\rm max}$	$\|{<}m{>}\|^{\rm QD}_{\rm min}$
0.0	2.8 (3.1) [2.8 (3.1)]	17.8 (13.8) [15.5 (12.0)]	53.0 (57.8) [46.4 (50.6)]	78.0 (69.4) [77.6 (69.1)]
0.02	3.8 (4.2) [3.7 (4.1)]	17.4 (13.5) [15.2 (11.7)]	52.0 (56.6) [45.5 (49.6)]	72.5 (64.1) [72.0 (63.7)]
0.04	4.8 (5.3) [4.6 (5.0)]	17.0 (13.3) [14.8 (11.5)]	50.9 (55.5) [44.5 (48.6)]	66.9 (58.7) [66.4 (58.4)]

Table 3: The maximal values of $|{<}m{>}|$ for the NH and the IH spectrum and the minimal values of $|{<}m{>}|$ for the IH and QD spectra (in units of 10^{-3} eV), obtained by using the best fit values of the solar and the atmospheric neutrino oscillation parameters and including 1 s.d. (3 s.d.) uncertainties of 5% (15%) on $\tan^2\theta_\odot$ and $\Delta m^2_\odot$, and of 10% (30%) on $\Delta m^2_{\rm A}$. Results for $\sin^2\theta = 0.0$, 0.02 and 0.04 are shown. Two values of $\Delta m^2_{\rm A}\,|_{\rm BF}$ are used: $\Delta m^2_{\rm A}\,|_{\rm BF} = 2.6\times10^{-3}$ eV ^{2} [2.0×10^{-3} eV ^{2}]. The results for the NH and IH spectra are obtained for $m_1 = 10^{-4}$ eV, while those for the QD spectrum correspond to $m_0 = 0.2$ eV.

In order to be possible to discriminate between the NH and the IH spectrum, the following inequality should be fulfilled [33]: $\zeta|{<}m{>}|^{\rm NH}_{\rm max} < |{<}m{>}|^{\rm IH}_{\rm min}$. The latter implies an upper limit on ζ. For the currently favored values of the neutrino oscillation parameters and $\sin^2\theta \lesssim 0.03$, the NH spectrum can be distinguished from the IH one even if $\zeta \sim 3$. If $\sin^2\theta \gtrsim 0.03$, $|{<}m{>}|^{\rm NH}_{\rm max}$ can be larger, as Table 3 illustrates, and somewhat smaller values of ζ could be required. Similarly, since $|{<}m{>}|^{\rm IH}_{\rm min} \sim \sqrt{\Delta m^2_{\rm A}}\,|\cos 2\theta_\odot|$, a shift of $\Delta m^2_{\rm A}$ to smaller values would lead to stronger constraints on ζ. In the "worst possible case" in which we allow a 3 s.d. error on $\Delta m^2_\odot$, $\Delta m^2_{\rm A}$ and $\tan^2\theta_\odot$, and use the best fit values of the solar neutrino oscillation parameters, $\Delta m^2_{\rm A} \; = 2.0\times10^{-3}$ eV ^{2} and $\sin^2\theta = 0.04$, it is necessary to have $\zeta < 2.3$.

The possibility to discriminate between the NH and the QD spectra leads to a less stringent condition on ζ than the condition permitting to distinguishing between the NH and the IH spectra: the former is satisfied even for values of ζ exceeding 3. The IH spectrum can be distinguished from the QD spectrum only if $\zeta \lesssim 1.5$ [6], unless additional information on neutrino masses is provided by the ^{3}H β-decay experiments and/or cosmological and astrophysical measurements.

[6]The upper bound on ζ varies from 1.1 to 1.6 according to the different values of $\Delta m^2_{\rm A}$ and $\sin^2\theta$ (see Table 3). The constraint is less stringent for smaller values of $\Delta m^2_{\rm A}$ and $\sin^2\theta$.

We update also the conditions on ζ and Δ which would permit to rule out, or establish, the NH, IH and the QD mass spectrum. The next generation of $(\beta\beta)_{0\nu}$-decay experiments are planned to reach a sensitivity of $|<m>| \sim 0.01 - 0.03$ eV . The QD mass spectrum can be ruled out if $\zeta < |<m>|^{\rm QD}_{\rm min}/((|<m>|_{\rm exp})_{\rm MIN} + \Delta)$. For the prospected sensitivity of $|<m>| \sim 0.01$ eV and $\Delta \sim 0.01$ eV , the requirement on ζ reads $\zeta < 3$. In the less favorable case in which $((|<m>|_{\rm exp})_{\rm MIN} + \Delta) \sim 0.05$ eV , an extremely accurate knowledge of the nuclear matrix elements, i.e. $\zeta \sim 1$, would be necessary. In order to establish that the spectrum is of the QD type, the following condition has to be fulfilled: $((|<m>|_{\rm exp})_{\rm MIN} - \Delta) \geq 0.2$ eV , as it implies that $m_0 \geq 0.2$ eV . This requirement is satisfied if, e.g., the measured $(|<m>|_{\rm exp})_{\rm MIN} \sim 0.3$ eV with an experimental error $\Delta \lesssim 0.1$ eV. The NH spectrum can be excluded provided the measured value of $|<m>|$ is larger than the predicted upper limit on $|<m>|$ for this type of spectrum, $(|<m>|_{\rm exp})_{\rm MIN} - \Delta > |<m>|^{\rm NH}_{\rm max}$. Given the expected sensitivities on $|<m>|$, this condition can be realized if, e.g., $(|<m>|_{\rm exp})_{\rm MIN}$ is at least 3-11 times larger than $|<m>|^{\rm NH}_{\rm max}$, i.e. $((|<m>|_{\rm exp})_{\rm MIN} \sim 0.015 - 0.035)$, and the experimental error amounts to at most $\Delta \sim 0.01 - 0.03$ eV . A larger measured value of $(|<m>|_{\rm exp})_{\rm MIN}$ would allow to exclude the NH spectrum even for larger values of Δ. The IH spectrum can be ruled out if $(|<m>|_{\rm exp})_{\rm MIN} - \Delta > |<m>|^{\rm IH}_{\rm max}$. For an experimental error on $|<m>|$, $\Delta \simeq 0.01;\ 0.03;\ 0.05$ eV , this condition is satisfied if $(|<m>|_{\rm exp})_{\rm MIN} > 0.06;\ 0.08;\ 0.1$ eV . Establishing that the spectrum is of the IH type is quite demanding and requires a measurement of $|<m>|$ with an error $\Delta \lesssim 0.015$ eV [7].

The possibility of establishing CP- violation in the lepton sector due to Majorana CP-violating phases has been studied in detail in ref. [34]. It was found that it requires quite accurate measurements of $|<m>|$ and of m_1, and holds only for a limited range of values of the relevant parameters. For the IH and the QD spectra, which are of interest for this analysis, the "just CP-violation" region [5] - an experimental point in this region would signal unambiguously CP-violation associated with Majorana neutrinos, is larger for smaller values of $\cos 2\theta_\odot$. As the present best fit values of $\Delta m^2_\odot$, $\Delta m^2_{\rm A}$ and especially of $\sin^2\theta_\odot$ are very similar to those used in [34], the conclusions reached in ref. [34] still hold. More specifically, proving that CP-violation associated with Majorana neutrinos takes place requires, in particular, a relative experimental error on the measured value of $|<m>|$ not bigger than $(15 - 20)\%$, a "theoretical uncertainty" in the value of $|<m>|$ due to an imprecise knowledge of the corresponding nuclear matrix elements smaller than a factor of 2, a value of $\tan^2\theta_\odot \gtrsim 0.55$, and values of the relevant Majorana CP-violating phases $(\alpha_{21},\ \alpha_{32})$ typically within the ranges of $\sim (\pi/2 - 3\pi/4)$ and $\sim (5\pi/4 - 3\pi/2)$.

4 Conclusions

In the present Addendum, we have updated the predictions for the effective Majorana mass in $(\beta\beta)_{0\nu}$-decay $|<m>|$ derived in ref. [1] by taking into account the implications of the recently announced results from the salt phase measurements of the SNO experiment. The combined analyzes of the solar neutrino data, including the latest SNO results, lead to new relatively stringent constraints on the solar neutrino mixing angle $\theta_\odot$: i) the possibility of $\cos 2\theta_\odot = 0$ is excluded at more than 5 s.d., ii) the best fit value of $\cos 2\theta_\odot$ is found to be $\cos 2\theta_\odot = 0.40$, and iii) at 95% C.L. one has $\cos 2\theta_\odot \gtrsim 0.22$. These new results firmly establish the existence of significant lower bounds on $|<m>|$ in the cases of IH and QD neutrino mass spectra, which in turn has fundamental implications for the searches for $(\beta\beta)_{0\nu}$-decay. Using, e.g., the 90% C.L. allowed ranges of the values

[7] Let us note that unless there is additional information on the type of neutrino mass hierarchy or on the range of allowed values of m_1, it will be in principle impossible to distinguish the case of IH spectrum from the one with partial normal hierarchy [5] ($\Delta m^2_\odot \equiv \Delta m^2_{21}$ and 0.01 eV $< m_1 < 0.2$ eV) using only a measurement of $|<m>|$, as the predicted allowed ranges of $|<m>|$ in the two cases overlap (see Figs. 1 and 2).

of the solar and atmospheric neutrino oscillation parameters one finds for the IH and QD spectra, respectively: $|<m>| \gtrsim 0.010$ eV and $|<m>| \gtrsim 0.043$ eV. The lower bounds obtained utilizing the 95% C.L. allowed values of the parameters do not differ substantially from those given above.

We have also updated the earlier results in [33, 34] on the possibilities i) to discriminate between the different types of neutrino mass spectrum (NH vs IH, NH vs QD and IH vs QD), and ii) to get information about CP-violation induced by the two Majorana CP-violating phases in the PMNS mixing matrix, if a value $|<m>| \neq 0$ is measured in the $(\beta\beta)_{0\nu}$-decay experiments of the next generation. The remarkable physics potential of the future $(\beta\beta)_{0\nu}$-decay experiments for providing quantitative information, in particular, on the type of the neutrino mass spectrum and on the CP-violation associated with Majorana neutrinos, can be fully exploited only if the values of the relevant $(\beta\beta)_{0\nu}-$decay nuclear matrix elements are known with a sufficiently small uncertainty.

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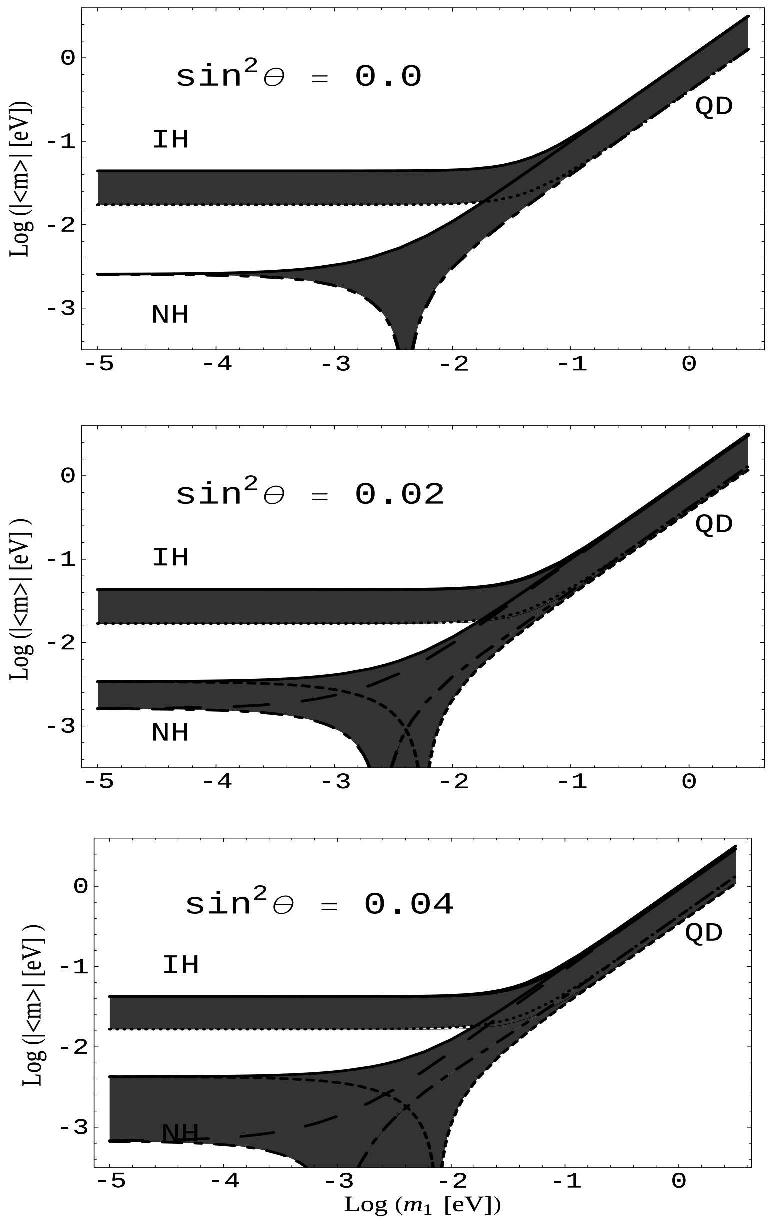

Figure 1: The dependence of $|<m>|$ on m_1 in the case of the LMA-I solution, for $\Delta m^2_{\odot} = \Delta m^2_{21}$ and $\Delta m^2_{\odot} = \Delta m^2_{32}$, and for the best fit values of the solar neutrino oscillation parameters found in ref. [16] and of $\Delta m^2_{\rm A}$ in ref. [27], and fixed value of $\sin^2\theta = 0.0$ (0.02) [0.04] in the upper (middle) [lower] panel. In the case of CP-conservation, the allowed values of $|<m>|$ are constrained to lie: for i) $\Delta m^2_{\odot} = \Delta m^2_{21}$ and the middle and lower panels (upper panel) - *a)* on the lower thick solid lines (on the lower thick solid line) if $\eta_{21} = \eta_{31} = 1$, *b)* on the long-dashed lines (on the lower thick solid line) if $\eta_{21} = -\eta_{31} = 1$, *c)* on the dash-dotted lines (on the dash-dotted lines) if $\eta_{21} = -\eta_{31} = -1$, *d)* on the short-dashed lines (on the dash-dotted lines) if $\eta_{21} = \eta_{31} = -1$; and for ii) $\Delta m^2_{\odot} = \Delta m^2_{32}$ (both panels) - *a)* on the upper thick solid line if $\eta_{21} = \eta_{31} = \pm 1$, *b)* on the dotted lines if $\eta_{21} = -\eta_{31} = \pm 1$. In the case of CP-violation, the allowed regions for $|<m>|$ cover all the gray regions. Values of $|<m>|$ in the dark gray regions signal CP-violation.

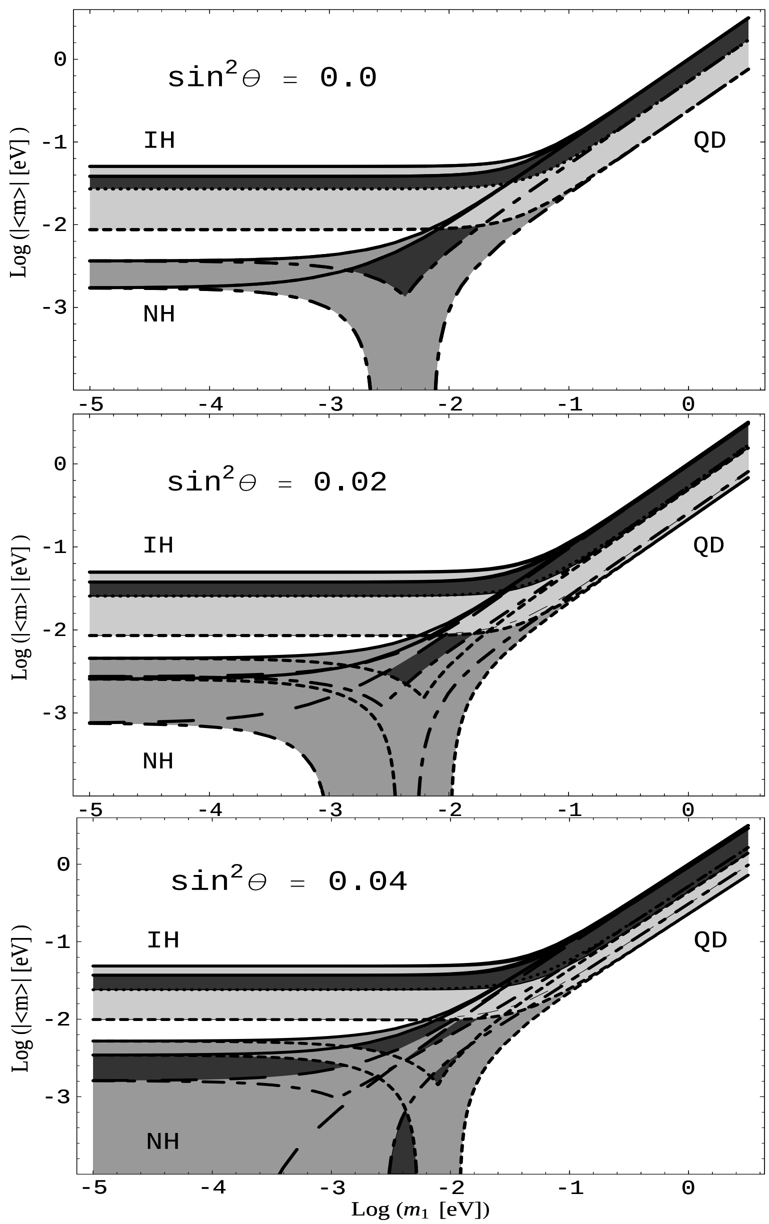

Figure 2: The dependence of $|<m>|$ on m_1 in the case of the LMA-I solution, for $\Delta m^2_\odot = \Delta m^2_{21}$ and $\Delta m^2_\odot = \Delta m^2_{32}$, and for the 90% C.L. allowed values of $\Delta m^2_\odot$ and $\sin^2\theta_\odot$ found in ref. [16] and of $\Delta m^2_{\rm A}$ in ref. [27], and a fixed value of $\sin^2\theta = 0.0(0.02)[0.04]$ in the upper (middle) [lower] panel. In the case of CP-conservation, the allowed values of $|<m>|$ are constrained to lie: for i) $\Delta m^2_\odot = \Delta m^2_{21}$ and the middle and lower panels (upper panel) - in the medium-gray and light-gray regions *a)* between the two lower thick solid lines (between the two lower thick solid lines) if $\eta_{21} = \eta_{31} = 1$, *b)* between the two long-dashed lines (between the two lower thick solid lines) if $\eta_{21} = -\eta_{31} = 1$, *c)* between the three thick dash-dotted lines and the axes (between the dash-dotted lines and the axes) if $\eta_{21} = -\eta_{31} = -1$, *d)* between the three thick short-dashed lines and the axes (between the dash-dotted lines and the axes) if $\eta_{21} = \eta_{31} = -1$; and for ii) $\Delta m^2_\odot = \Delta m^2_{32}$ and the middle and lower panels (upper) - in the light-gray regions *a)* between the two upper thick solid lines (between the two upper thick solid lines) if $\eta_{21} = \eta_{31} = \pm 1$, *b)* between the dotted and the thin dash-dotted lines (between the dotted and the thick short-dashed lines) if $\eta_{21} = -\eta_{31} = 1$, *c)* between the dotted and the upper thick short-dashed lines (between the dotted and the thick short-dashed lines) if $\eta_{21} = -\eta_{31} = -1$. In the case of CP-violation, the allowed regions for $|<m>|$ cover all the gray regions. Values of $|<m>|$ in the dark gray regions signal CP-violation.